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ANALYSIS FOR SHEET PILE RETAINING WALLS

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ANALYSIS FOR SHEET PILE RETAINING WALLS

F. E. Richart, Jr., A. M. ASCE[1]

SYNOPSIS

A method for analyzing sheet pile retaining walls is developed by considering the piling to be cut at the dredge line. The shear, moment, and backfill pressures due to the material above the dredge line are computed and then are applied as loads to the portion of the pile which is embedded. This embedded length of the pile is then treated as a beam on an elastic foundation in which the stiffness of the elastic foundation is assumed to increase linearly with depth below the dredge line, and to be zero at that point. This type of foundation corresponds roughly to the behavior of sands. Graphs have been prepared which show the variations in deflection and moment in this section of the pile due to a concentrated shear or moment at the dredge line, or a uniform pressure along the embedded length. Using this information, a procedure for analyzing a chosen sheet pile installation is presented and the results for two examples are compared with those given by the "classical" procedures for such structures. The results of this study show the importance of the relative flexibilities of the pile section and foundation.

INTRODUCTION

The recent paper by Karl Terzaghi,[2] Hon. M.A.S.C.E., has emphasized the need for including a consideration of the wall flexibility in the design procedures for sheet pile retaining walls. Tests by P. W. Rowe[3] and by G. P. Tschebotarioff[4] have indicated a reduction of maximum bending moment and anchor load to values considerably below those computed from the classical "free earth support" procedure, as a result of increasing the wall flexibility. In addition to considering wall flexibility, Rowe's tests also included an evaluation of the density of the material into which the model wall was embedded. Consequently, Terzaghi proposed that curves based on Rowe's tests could be utilized during the design of a particular sheet pile wall. That is, the curves permit an estimate of the reduction below the "free earth support" values of bending moment and anchor pull caused by the wall flexibility and condition of the sand in the passive pressure zone.

However, Rowe's curves were based on laboratory tests of dry materials which had approximately the same density above and below the dredge line. The particular parameters investigated were: (a) the wall flexibility number,

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1. Prof. of Civ. Eng., Univ. of Florida, Gainesville, Fla.
2. "Anchored Bulkheads," by Karl Terzaghi. Proceedings Separate No. 262 ASCE, September 1953.
3. "Anchored Sheet Pile Walls," by P. W. Rowe, Proceedings, Institution of Civil Engineers, Vol I, January, 1952, pp 27-70.
4. "Final Report, Large Scale Earth Pressure Tests with Model Flexible Bulkheads," by G. P. Tschebotarioff, Princeton University, Princeton, N.J., 1949.

$\rho = \frac{(H')^4}{EI} \frac{ft.^4}{lb.in^2}$ per foot, where H' is the total pile length in feet, (b) the anchor point location defined as the distance $\beta H'$ from the top of the pile, (c) the length of pile embedded below the dredge line, (d) the amount of surcharge, and (e) the type and density of material which was supported by the wall. Thus considerable judgment is required in order to extrapolate this test information for use in actual pile design. However, such tests establish criteria for evaluating existing design procedures and until data from tests on prototype structure are reported, they are the most reliable information available.

The reduction in bending moment and anchor pull found in the tests[2,3] was due to the partial fixity applied to the embedded length of the pile by sand pressures. Since the amount of fixity depends upon the relative stiffnesses of the sand and wall in this region, the theory of subgrade reaction can be utilized to estimate the relative importance of each quantity under several types of loading conditions.

Previous analyses of this type have been made by P. Baumann,[5] I. Rifaat,[6] H. Blum,[7] and others, to study the behavior of single piles and sheet pile walls.

This paper is an attempt to extend the previous analytical studies of sheet pile walls. In order to make the results of rather tedious computation more usable, graphs have been prepared for coefficients of deflection, and moment for each of three types of loading on the embedded portion of the pile. Thus, the solution for any given pile problem can be obtained by superposing these component solutions.

Notation:

The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically, for convenience of reference, in the Appendix.

ASSUMPTIONS

The subgrade material is assumed to behave as a series of independent elastic springs. Furthermore, the rigidity of these springs is assumed to vary linearly with depth, and to have a value of zero at the surface. This assumption appears valid within the range of depths which are normally considered for sheet pile walls embedded in sand. Loose sands have shown a linear relations between initial tangent modulus and lateral confining pressure for pressures up to about 40,000 lb/ft^2. Dense sands exhibit a curved relation between initial tangent modulus and confining pressure, but up to about 4000 lb/ft^2 this portion of the curve can be approximated by a straight line. Since an over burden height of about 40 ft. of dry sand or 60 ft. of submerged sand is required to produce this pressure, the usual embedded lengths of sheet piling act upon sand which exhibits an approximately linear relation between deformation and pressure for small deformations.

For the analysis of a specific sheet pile wall, the structure is cut at the dredge line and the subgrade theory is used to analyze the behavior of the

5. "Analysis of Sheet-Pile Bulkheads," by Paul Baumann, Trans. A.S.C.E., Vol.100, 1935, pp 707-797.
6. "Die Spundwand als Erddruckproblem," by I. Rifaat, Mitt. Inst. Baustatik No. 5 Zürich, 1935.
7. "Beitrage zur Berechuung von Bohlwerken," by H. Blum, W. Ernst and Son, Berlin, Germany, 1951.

embedded length of pile due to the applied loads. These loads consist of a moment and a concentrated shear force acting at the dredge line as well as the horizontal pressure along the embedded length produced by the overburden pressure.

This horizontal pressure is the active earth pressure generated by the weight of the backfill and the surcharge, and it is assumed that this pressure is constant along the embedded length of the pile. Before any backfilling or dredging is done, the embedded portion of the pile is acted upon by the earth pressure at rest on both sides. As backfilling progresses the piling moves slightly outward. Thus on the inside face the pressure consists of the active earth pressure exerted by the original embedment depth plus the active earth pressure exerted by the backfill depth.

On the outside face the original earth pressure at rest remains, but an increase in pressure due to the passive resistance of the soil is generated. Figs. 1 (a) and 1 (b) show those conditions of pressure distribution. In this analysis, it is assumed that the active pressure on the inside face is equal to the earth pressure at rest on the outside face so that they cancel each other. This approximation contributes a conservative error to the analysis since it ignores the restraint offered to the pile by the difference between the earth pressure at rest and the active earth pressure. This leaves only the uniform value of active earth pressure due to the backfill to act as a load and the passive earth pressure as the reaction.

Analysis of Component Solutions

If the sand is considered to behave as an elastic medium, the relation between the wall displacement and the horizontal reaction pressure it develops can be expressed as,

$$p_h = k_h\, y \tag{1}$$

In Eq. (1), k_h is the coefficient of horizontal subgrade reaction. For this study, k_h is assumed to vary linearly with depth, or,

$$k_h = n_h\, z \tag{2}$$

As a further designation, if "C" is the value of the coefficient of subgrade reaction at the bottom end of the pile, then

$$n_h = \frac{C}{D} \tag{3}$$

where D is the embedded length of the pile. Then by substituting Eqs. (2) and (3) in Eq. (1),

$$p_h = \frac{C}{D}\, z\, y \tag{4}$$

End Loads Only

In order to analyze the sheet pile wall, it is convenient to consider a section of the wall of length D, and one unit wide. Thus, the solution for a beam resting on an elastic foundation becomes applicable. Hayaski[8] and later

8. "Theorie des Trägers auf elastischer Unterlag," by K. Hayashi, Julius Springer, Berlin, 1921.

Hetényi[9] have presented solutions for beams of uniform EI resting on a foundation whose compressibility varies linearly along the length of the beam according to the expression,

$$k_x = k_A - bx \quad . \tag{5}$$

In Eq. (5), k_A lb/in^2 and b lb/in^3 are constants, and x is the distance along the beam from a fixed point A. For the particular case of the subgrade coefficient which is zero at the surface (x=D) and equal to C at the bottom of the pile (x=0), Eq. (3) becomes,

$$k_h = C - C\,\frac{x}{D} = C\,\frac{(D-x)}{D} = \frac{Cz}{D}, \tag{6}$$

where z is the distance from the top of pile. Using Eq. (6) in the differential equation of bending for a segment of the beam, the following equation is obtained:

$$EI\,\frac{d^4y}{dx^4} + \frac{(D-x)}{D}\,Cy = 0 \tag{7}$$

By introducing a new variable,

$$\xi = \frac{D-x}{D} = \frac{z}{D} \tag{8}$$

we have

$$\frac{d^4y}{dx^4} = \frac{d^4y}{d\xi^4}\left(\frac{d\xi}{dx}\right)^4 = \frac{d^4y}{d\xi^4}\,\left(\frac{1}{D^4}\right) \tag{9}$$

and thus Eq. (7) takes the form,

$$\frac{d^4y}{d\xi^4} + \alpha\xi y = 0 \tag{10}$$

where

$$\alpha = \frac{CD^4}{EI} \quad . \tag{11}$$

α is a dimensionless quantity which relates the stiffness of the soil (C) to the stiffness of the sheet pile section $\left(\frac{EI}{D^4}\right)$, and consequently it is designated the "flexibility coefficient" in this analysis.

Solution of Eq. (10) by means of the recursion formula gives the general solution for y in terms of four series, y_1, y_2, y_3 and y_4 as,

$$y = c_1y_1 + c_2y_2 + c_3y_3 + c_4y_4 \tag{12}$$

The component series involved in the general solutions are,

$$\begin{aligned}
y_1 &= 1 - \frac{\alpha\xi^5}{5!} + \frac{6\alpha^2}{10!}\xi^{10} - \frac{6\cdot11\alpha^3}{15!}\xi^{15} + \frac{6\cdot11\cdot16\alpha^4}{20!}\xi^{20} - \cdots \\
y_2 &= \xi - \frac{2\alpha\xi^6}{6!} + \frac{2\cdot7\alpha^2}{11!}\xi^{11} - \frac{2\cdot7\cdot12\alpha^3}{16!}\xi^{16} + \frac{2\cdot7\cdot12\cdot17\alpha^4}{21!}\xi^{21} - \cdots \\
y_3 &= \frac{\xi^2}{2!} - \frac{3\alpha}{7!}\xi^7 + \frac{3\cdot8\alpha^2}{12!}\xi^{12} - \frac{3\cdot8\cdot13\alpha^3}{17!}\xi^{17} + \frac{3\cdot8\cdot13\cdot18\alpha^4}{22!}\xi^{22} - \cdots \\
y_4 &= \frac{\xi^3}{3!} - \frac{4\alpha}{8!}\xi^8 + \frac{4\cdot9\alpha^2}{13!}\xi^{13} - \frac{4\cdot9\cdot14\alpha^3}{18!}\xi^{18} + \frac{4\cdot9\cdot14\cdot19\alpha^4}{23!}\xi^{23} - \cdots
\end{aligned} \tag{13}$$

9. "Beams on Elastic Foundation" by M. Hetényi, University of Michigan Press, Ann Arbor, 1946.

From the above expression for the deflection, the expressions for slope, moment, and shear at any point on the beam can be obtained by successive differentiations. That is,

$$\theta = \frac{dy}{dx} = \frac{dy}{d\xi} \cdot \frac{d\xi}{dx} = \frac{1}{D} \frac{dy}{d\xi} = \frac{K_\theta}{D}$$

$$M = -EI \frac{d^2y}{dx^2} = \frac{-EI}{D^2} \frac{d^2y}{d\xi^2} = \frac{-EI}{D^2} K_m \tag{14}$$

and

$$V = -EI \frac{d^3y}{dx^3} = \frac{EI}{D^3} \frac{d^3y}{d\xi^3} = \frac{EI}{D^3} K_v$$

The boundary conditions at $\xi = 0$ (dredge line) and at $\xi = 1.0$ (bottom of pile) permit evaluation of the constants in Eq. (12).

In Eq. (14) the various derivatives of y with respect to the dimensionless quantity ξ are represented as coefficients, K_θ, etc. These expressions for K have been evaluated for the range of ξ from 0 to 1.0 and for a range of α from 50 to 10,000 for end moment only, and for end shear only; the results are shown on Figs. 2 (a), 3, 4, 6, and 7.

Uniform Load Only

When a uniformly distributed load is carried by a beam resting on the type of elastic foundation previously described, Eq. (7) becomes,

$$EI \frac{d^4y}{dx^4} + \frac{C(D-x)}{D} y = p_o \tag{15}$$

By introducing the variable defined in Eq. (8) and rearranging, Eq. (15) becomes,

$$\frac{d^4y}{d\xi^4} + \alpha\xi y = \frac{D^4}{EI} p_o = \frac{\alpha p_o}{C} = \text{Const.} \tag{16}$$

Solution of Eq. (16) is obtained by adding to the complementary function (Eq. (12) , a particular solution,

$$y_p = \frac{p_o}{C}\left[\frac{\alpha\xi^4}{4!} - \frac{5\alpha^2}{9!}\xi^9 + \frac{5\cdot10\alpha^3}{14!}\xi^{14} - \frac{5\cdot10\cdot15\alpha^4}{19!}\xi^{19} + \cdots\right] \tag{17}$$

so that the final solution for deflections is given by,

$$y = c_1y_1 + c_2y_2 + c_3y_3 + c_4y_4 + y_p \tag{18}$$

For the case of uniformly distributed load only, with no moment or shear at $\xi = 0$ (dredge line), both c_3 and c_4 are zero and Eq. (18) is simplified somewhat.

The coefficients for deflection (K_y) and slope (K_θ) of the pile at the dredge line ($\xi = 0$) are given on Fig. 2 (b), and the coefficients for deflection, and moment, are shown on Figs. 5 and 8 all for values of α from 50 to 10,000.

Possible Values of the Coefficient "C"

Before the results of the mathematical analysis outlined in the previous section can be used, it is necessary to evaluate at least approximately this "coefficient of compressibility" of the soil "C."

The test results reported by Terzaghi[10] have been used as a basis for

10. "Large Retaining Wall Tests, I, Pressure of Dry Sands," by Karl Terzaghi, Engineering News-Record, Vol.112, 1934, pp 136-140.

computing values of C for loose sand, dense sand, and dense sand which had been disturbed by wall movement. The latter sequence of wall movement involved rotation of a rigid wall about its base, (a) into the sand and back to the original position (b) outward from the original position and back, and (c) again into the sand. It is this wall movement (c) which is considered to act upon "disturbed" sand. For sheet pile walls, the "working" back and forth of the pile near the dredge line would loosen dense sand, and tend to compact loose sand, so that an equilibrium condition would eventually be reached which would correspond roughly to the disturbed condition above.

The test information, given in Ref. 9, relating the lateral pressure coefficient to the lateral movement of the wall was used with Eq. 4 in order to establish the values of the coefficient, C, which are given in Column 5 of Table I. Since the embedded length of a sheet pile bends as well as rotates about some point near the tip, the distribution of passive pressures along this length is different from that along a rigid wall of comparable height. Consequently, it is assumed that for a preliminary analysis of a specific problem it will be satisfactory to determine the coefficient of subgrade reaction at the bottom of a given pile by the expression,

$$C_n = \frac{D}{D^1} C \tag{19}$$

in which C and D^1 are given in Table I, Columns 5 and 3, respectively, and D is the embedded length (in inches) of the particular pile. The subscript n (corresponding to the number in feet) is used to denote the depth associated with a particular value of C. Thus, Column 6 of Table I gives the values of C_{10} which correspond to an embedded pile depth of 10 ft.

General Procedure for Analysis

In order to use the coefficients given on Figs. (3) through (8), it is first necessary to determine the value of moment, shear, and overburden pressure which act at the dredge line.

For cantilever piles, this is relatively easy since both moment and shear can be determined directly by statics. For anchored sheet pile walls, however, the problem is statically indeterminate so that one or more of the boundary conditions must be incorporated into the problem. For a bulkhead with a single anchor point it is convenient to consider the deflection of the anchor point to be zero as indicated in the equation,

$$y_{\text{anchor}} = y_s - y_{Ap} + y_{oM} + y_{oV} + y_{oP} + L(\theta_{oM} + \theta_{oV} + \theta_{oP}) = 0 \tag{20}$$

The subscript "o" in Eq. (21) designates the deflection at the dredge line due to moment, shear or pressure.

These component deflections are shown on Fig. (9). In Fig. 9 (b), the wall is considered to be a cantilever above the dredge line and to be loaded by the active pressure of the soil. The deflection at the anchor point produced by these soil loads is designated by y_S, and is readily computed. The writer has found the procedure developed by Newmark[11] to be particularly useful for this type of computation, since the shear, moment, and deflections are all obtained directly. Also, the calculation for deflection is not complicated by an irregular load distribution or by a variation in moment of inertia of this portion of the pile, as would occur for a pile section reinforced by cover plates. Since

11. "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads," by N. M. Newmark. Transactions ASCE Vol.108, 1943, pp1161-1188.

the value of the anchor pull is unknown, the moment and shear at the dredge line may be written as,

$$M_o = M_s - A_p l,$$

and

$$V_o = V_s - A_p \qquad (21)$$

where M_s and V_s are the moment and shear at the dredge line produced by the soil load above the dredge line.

Eq. (20) now contains two unknown quantities, α and A_p. Since α is a flexibility coefficient composed of the soil compressibility, C, the depth of embedment D, and the E and I of the pile section, various arrangements of these quantities can yield the same value of α.

Two examples of anchored bulkheads were chosen to illustrate the use of the procedure. The dimensions and loading conditions of each are shown in Fig. 10. Example A represents a bulkhead driven into clean, cohesionless sand and backfilled with the same material. Example B is similar to the bulkhead of Example A except that an 8 ft. layer of soft clay is situated just below the anchor level and a 500 lb./ft.^{2} surcharge acts on the surface of the backfill. Example B represents an extreme case which would probably not occur in practice since the clay would be removed, at least in the vicinity of the bulkhead, and a "sand dike" used to reduce the lateral pressure on the bulkhead.

Since the maximum moment indicated by the "equivalent beam" procedure[12] was 92,000 ft. lb./ft. for Example A and 118,000 ft. lb./ft. for Example B, by using an allowable stress of 23,000 psi the section moduli required are 48.0 in.^{3}/ft. and 61.5 in.^{3}/ft., respectively. Thus section MZ-38 (46.8 in.^{3}/ft.) can be used for Example A, and MZ-38 reinforced with cover plates 4 3/4 in. by 3 8 in., (60.4 in.^{3} ft.) will be satisfactorily for Example B. The reinforcement for Example B would not be carried below the dredge line so the value of EI for the embedded portion of the pile will be the same for both examples.

Since the value C_n depends upon the required length of embedment (D, inches) its value can be obtained by multiplying the value in Column 5, Table I by the ratio $\frac{D}{D1}$. For the examples chosen, the sand was assumed to be in the "disturbed" condition.

Three embedment depths, 0.4H (16 ft), 0.55H (22 ft), and 0.70H (28 ft) were used for the bulkheads of Examples A and B to give end restraints covering the range from free to fixed. Table II indicates the procedure for computing the anchor pull, and the moment and shear which act on the pile at the dredge line. Values of the deflection and slope coefficients at the dredge line are determined from Fig. 2 for each type of loading and for each particular value of the flexibility coefficient, α. Then the component deflections are obtained by multiplying these coefficients by the structural constants and the values of the applied loads. Since these loads are given as indicated in Eq. (21), the final expression (Eq. 20) for deflection at the anchor point involves only one unknown, A_p, so that it can be evaluated directly. Once the value of anchor pull is obtained, the moment and shear at the dredge line are found from Eq. (21). Then by use of this shear and moment, and the uniform pressure acting along the embedded depth D, the deflection curve for each case can be computed by use of the coefficients of Figs. 3, 4, 5, and finally, reaction pressures are computed by use of Eq. (4). Figs. 11 (a) and 11 (b) show the variation in

12. "Theoretical Soil Mechanics" by K. Terzaghi. J. Wiley and Sons, 1943, p. 225.

deflection and pressure which result from the three depths of embedment for Example A. In each case the value "C_n" was determined by the relation $\frac{420}{58.8}$ which denotes the same compressibility coefficient of the sand at each elevation for all three cases.

A further comparison was made by considering only the 28 ft. depth of embedment but using "C_{28}" values of 15,72 and 240 lb/in^3. The latter value corresponds to the "disturbed" condition and the smaller values are for loose sands. These particular values of C_{28} were chosen so as to give the same three values of α of 275,1330, and 4450 that were produced by the three different depths of embedment for C_{28} = 240 lb/in^3. The procedure for determining anchor pull, bending moment, etc, is similar to that outlined in Table II. Deflections and pressures for this set of conditions are shown on Fig. 11 (c) for Example A. The curves corresponding to the results obtained for Example B are similar.

Discussion of Results

By breaking the analysis of sheet pile retaining walls into components, the contribution of each type of load can be evaluated. Fig. 11 (a) illustrates the behavior of the embedded section of the wall due to the uniform lateral pressure exerted by the backfill. For the three values of D considered for Example A, it should be noted that the entire embedded length moves outward from its original position. The shorter wall also tends to rotate as a rigid body while the longer walls bend so as to approximate the position of the short wall in the vicinity of the dredge line; consequently, the pressure distribution curves are similar down to about 14 ft. below the dredge line. Below this depth the pressure is determined by the deflection at each point, which in turn is governed by the relative stiffnesses of the pile and sand. As the pile penetrates deeper, its stiffness becomes less important with respect to that of the sand, and the restraining pressure tends to become equal to the loading pressure.

Similar curves result from application of the end moment and end shear which act at the dredge line. These can be determined by use of Figs. 3 and 4 and Eq. (4). Since the resultant shear acts to displace the pile outward and the moment tends to rotate it inward, for an anchored bulkhead, these two effects tend to cancel. However, in most cases the net moment at the dredge line is small so that the effect of the shear is predominant. As indicated on Figs. 11 (a) and 11 (b) the combined effect of shear and moment produced an outward deflection at the dredge line of about 0.7 inches as compared to 0.3 for the uniform pressure alone.

From a consideration of these deflections, the procedure for analysis as outlined in Table II might be varied to account for movement of the anchor point. If the anchor point has very little motion, then the procedure is as illustrated in Table II. However, if the anchor point is assumed to move as much as does the bulkhead at the dredge line, then the anchor pull value is lowered somewhat. For this condition, the anchor pull value for D = 16 ft, Example A, is 11,300 lb/ft instead of the 11,600 lb/ft originally computed. Thus it is seen that such a modification in the method of analysis is unnecessary.

The design criteria of anchor pull, maximum bending moment, and maximum passive pressure are all affected by variations in factors of the dimensionless flexibility coefficient $\alpha\ (\frac{CD^4}{EI})$. However, for the condition of equal

soil stiffness, but with three depths of penetration, the maximum bending moment was nearly constant. For the case of constant embedment depth but variable soil stiffness, a decrease of C_{28} to 6.25 percent of its original value caused an increase in anchor pull of about 15 percent and an increase of maximum bending moment of about 40 percent, for Example A. Values of anchor pull, maximum moment, depth of embedment, and maximum value of the coefficient of passive earth pressure are given in Table III as computed by the present method of analysis and by the conventional procedures.

The conventional procedures used for analysis of the example bulkheads were (a) the free earth support method, (b) the equivalent beam method, and (c) the elastic line method. Values of 3.0 and 6.0 were used as the coefficient of passive earth pressure in computations by each method.

The values of anchor pull and bending moment computed by the subgrade theory were found to be roughly 10 and 20 percent lower, respectively, than those computed by the elastic line method using $K_p = 6$. These reductions indicate the amount by which the flexibility of the pile affects the results in this particular case. However, these reductions of anchor pull and moment result from an elevation of the center of pressure of the passive pressure zone as indicated on the diagrams in Fig. 11. As this center of pressure moves upward, the pressure distribution curve exhibits high pressure values near the dredge line. The dotted lines on Fig. 11 (a), (b), and (c) which define the initial tangents of these curves correspond to values of the coefficient of passive earth pressure. Computed values of K_p ranging from 6 to 17 were obtained from analysis of the example problems, as indicated in Table III and Fig. 11.

These high computed values of K_p are very likely unrealistic values since they exceed considerably the lower limiting values of K_p of 9.0 and 5.0 for dense and loose sands, respectively, which were suggested by Terzaghi.[1] However, in an actual situation, if the sand at the surface was overloaded, the necessary reaction would be developed at a slightly greater depth by an increase in pressure in this region. This, of course, results in a slight increase in both anchor pull and bending moment.

CONCLUSIONS

In the preceding pages a method for analyzing sheet pile installations has been devised which incorporates a numerical procedure for determining the shear, moment, and pressure at the dredge line and uses the theory of subgrade reaction for the embedded pile length. This procedure permits computations of anchor pull and maximum bending moment in the pile which are comparable to those obtained by the conventional methods.

As illustrations of the procedure, two examples were studied. The computed values of anchor pull and maximum bending moment were slighly lower than those given by the conventional methods, due to the high values of passive pressure developed near the dredge line. Since these high pressure values imply values of the coefficient of passive earth pressure which are unrealistic in some cases, it is necessary to use caution in the application of the analytical results obtained from this procedure.

Finally, the success of such a method of analysis depends upon the reliability of the material constants involved. In this case it is the coefficient of compressiblity (C lb/in^3) of the sand which is the critical quantity, although variations of C by a factor of three or four times had little effect on the results of the examples studied. Several values of C have been estimated from the results of a few tests in which the passive resistance of sands were

reported. In anticipation of more test results becoming available in the future, particularly those for full size sheet pile walls, graphs have been prepared for the deflection and moment coefficients caused by end shear or moment, or uniform pressure on the embedded pile length. Thus "C" values can be derived from bending moment curves or deflection curves from such tests.

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APPENDIX - LIST OF SYMBOLS

The following letter symbols conform essentially with ASCE Manual of Engineering Practice No. 21 ("Soil Mechanics Nomenclature").

A_p	anchor pull
b	constant
c_i	constants
C	coefficient of soil compressiblity at the bottom of a pile (force/length^3)
Cn	coefficient of soil compressiblity at the bottom of a pile "n" ft long
D	depth of sheet pile penetration (inches)
D.L.	dredge line
E	the modulus of elasticity
H	height of sheet pile above the dredge line
H'	total length of sheet pile
I	the moment of inertia of the cross section of a sheet pile
k_A	constant
k_h	coefficient of horizontal subgrade reaction at any depth
k_x	elastic foundation reaction at a point x
K_A	coefficient of active earth pressure
K_o	pressure coefficient of earth at rest
K_p	coefficient of passive earth pressure
K_{yM}, K_{yV}, K_{y_p}	deflection coefficients for the embedded portion of the pile for the conditions of end moment only, end shear only, and uniform pressure, respectively
$K_{\theta M}, K_{\theta V}, K_{\theta p}$	slope coefficients for the embedded portion of the pile for the conditions of end moment only, end shear only, and uniform pressure, respectively

K_{MM}, K_{Mv}, K_{Mp}	moment coefficients for the embedded portion of the pile for the conditions of end moment only, end shear only, and uniform pressure, respectively
L	distance between the dredge line and anchor point
m_h	rate of increase of k_h with depth
M	bending moment in the sheet pile at any location
M_o	bending moment in the sheet pile at the dredge line ($M_o = M_s - A_p \cdot L$)
M_{Mo}	bending moment in the embedded length of sheet pile caused by Mo.
M_{po}	bending moment in the embedded length of sheet pile caused by the uniform pressure, p_o
M_{Vo}	bending moment in the embedded length of sheet pile caused by V_o
M_s	bending moment in the sheet pile at the dredge line, considering the pile fixed at the dredge line and supporting the soil loads as a cantilever beam
p	horizontal pressure on the sheet pile
p_o	horizontal pressure on the sheet pile at the dredge line
p_h	horizontal reaction pressure due to passive earth resistance
V	shear in the sheet pile at any location
V_o	shear on the sheet pile at the dredge line, ($V_o = V_s - A_p$)
V_s	shear on the sheet pile at the dredge line due to the soil loads only
x	distance from the tip of the pile
y	horizontal movement or deflection of the pile at any elevation
y_i	component series
y_{Mo}, y_{Vo}, y_{po}	deflection of the pile at any elevation caused by the moment, or shear at the dredge line, or the uniform pressure
y_{Ap}	deflection at the anchor point caused by the anchor pull acting on the pile which is considered to be a cantilever above the dredge line
y_s	deflection at the anchor point caused by the soil loads acting on the pile which is considered to be a cantilever above the dredge line
Z	distance below the dredge line
α	flexibility coefficient ($\alpha = \frac{cD^4}{EI}$)
β	ratio of the distance from the top of the pile to the anchor point to the total pile length

γ	density of the sand
y_p	component series for deflection due to p_o only
θ	slope ($\theta = \frac{dy}{dx}$)
$\theta_{oM}, \theta_{oV}, \theta_{op}$	slope at the dredge line of the embedded portion of the pile loaded by M_o, V_o, or p_o
ρ	Rowe's wall flexibility number ($\rho = \frac{(H')^4}{EI}$ $\frac{ft^4}{lbin^2}$ per foot)
ξ	the ratio of the distance below the dredge line to the total embedment depth ($\xi = \frac{z}{D} = \frac{D-x}{D}$)

TABLE I

Approximate Values for the "Coefficient of Compressibility" (C) of Sands

1	2	3	4	5	6
Density of Sand lb/ft^3	Type Wall	Height of Wall(D'in) in contact with the Sand	Conditions of Sand	C lb/in^3	Coefficient at Base of 10ft.wall $C_{10} = \frac{120}{D'} C$
90.1	Rigid, Rotating about Base	62.4	Loose	12	23
100		58.8	Compacted	216	441
100		58.8	Disturbed	42	86

NOTE: The values of C given in Table I were computed from test data reported by K. Terzaghi (Ref. 9) for which the value of the coefficient of lateral earth pressure varied from about 0.2 to 2.2.

TABLE II

DETERMINATION OF SHEAR AND MOMENT AT THE DREDGE LINE (For C = 86. $\frac{D}{120}$ lb./in.^{2}, Example A)

Embedment Depth	D=192 (16)	D=264 (22)	D=336 (28)	Units in. (ft.)	Notes
EI	6.880×10^8	6.880×10^8	6.880×10^8	$\frac{\text{lb. in.}^2}{\text{in.}}$	
$\frac{D}{EI}$	$\frac{2.7910}{10^7}$	$\frac{3.8374}{10^7}$	$\frac{4.8840}{10^7}$	$\frac{\text{in.}^2}{\text{lb. in.}^2}$	
$\frac{D^2}{EI}$	$\frac{5.3585}{10^5}$	$\frac{1.0131}{10^4}$	$\frac{1.6410}{10^4}$	$\frac{\text{in}}{\text{lb.}}$	
$\frac{D^3}{EI}$	$\frac{1.0228}{10^2}$	$\frac{2.6745}{10^2}$	$\frac{5.5138}{10^2}$	$\frac{\text{in.}^2}{\text{lb.}}$	
$\frac{D^4}{EI}$	1.9754	7.0607	18.526	$\frac{\text{in.}^3}{\text{lb.}}$	
C	140	190	240	lb./in.^{2}	
$\alpha = \frac{CD^4}{EI}$	275	1330	4450		
$K_{y_{cM}}$	0.187	0.090	0.056		From Fig. 2(a)
K_{yov}	0.092	0.032	0.0157		From Fig. 2(a)
K_{yop}	6.85	8.60	11.0		From Fig. 2(b)
K_{cM}	0.595	0.415	0.327		From Fig. 2(a)
K_{ov}	0.187	0.090	0.056		From Fig. 2(a)
K_{op}	8.60	14.8	24.4		From Fig. 2(b)
p_o(psi)	6.875	6.875	6.875	lb./in.^{2}	
p_o/C	0.0498	0.0364	0.0285	in.	
y_s	$13.585 - 0.01838^x$	$13.585 - 0.01838^x$	$13.585 - 0.01838^x$	in.	
y_{oM}	$3.1474 - 0.003367^x$	$2.8639 - 0.003064^x$	$2.8855 - 0.003088^x$	in.	$y_{cM} = K_{y_{cM}}\left(\frac{MD^2}{EI}\right)$

TABLE II (Cont'd)

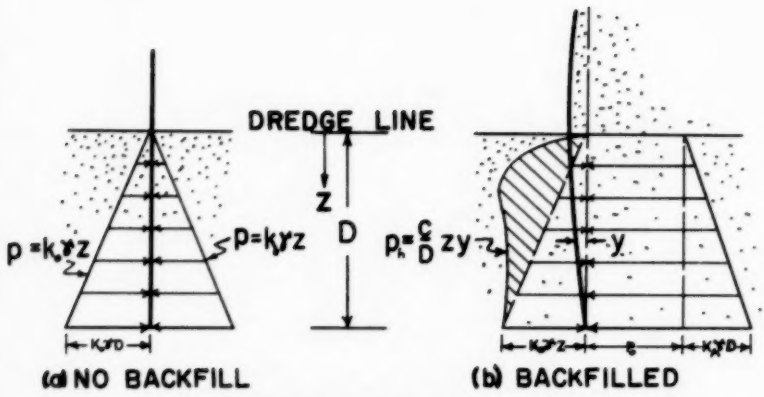
Embedment Depth	D=192 (16)	D=264 (22)	D=336 (28)	Units inches (ft.)	Notes
y_{ov}	1.7385 -0.000947[x]	1.5719 -0.000856[x]	1.5900 -0.000866[x]	in.	$y_{ov} = K_{y_{ov}} (\frac{VD^3}{EI})$
y_{op}	0.3413	0.3129	0.3138	in.	$y_{op} = K_{y_{op}} \frac{P_o}{C}$
$\theta_{cm} L$	17.5254 -0.01875[x]	16.8071 -0.01798[x]	16.8551 -0.01803[x]	in.	$\theta_{cm} L = K_{\theta_{cm}} (\frac{VD}{EI}) . L$
$\theta_{ov} . L$	6.1839 -0.00337[x]	5.6269 -0.00306[x]	5.6712 -0.00309[x]	in.	$\theta_{ov} . L = K_{\theta_{ov}} (\frac{VD^2}{EI}) . L$
$\theta_{op} . L$	0.7498	0.6853	0.6961	in.	$\theta_{op} . L = K_{\theta_{op}} \frac{P_o}{CD} . L$
Total Deflection	43.2713 -0.04814[x]	41.4530 -0.04334[x]	41.5967 -0.04345[x]	in.	
A_p	965.58 11,600	956.46 11,500	957.26 11,500	lb/in lb/ft	
$M_{DL} = M_s - A_p L$	-10,700	-7900	-7900	ft.lb./ft. (in.lb./in.)	
$V_{DL} = V_s - A_p$	871 10,450	880 10,560	880 10,560	lb./in. lb./ft.	

x These values are multiplied by A_p.

TABLE III

COMPARISON OF ANALYTICAL RESULTS FOR THE EXAMPLE BULKHEADS

Procedure	Example A				Example B			
	D (ft)	Ap (lb/ft)	M max (ftlb/ft)	Kp max	D (ft)	Ap (lb/ft)	M max (ftlb/ft)	Kp max
1. Subgrade Theory								
(a) "Disturbed"	16	11,600	70,000	17.0	16	24,400	97,000	19.0
(b) Sand	22	11,500	69,000	17.0	22	24,300	96,000	19.0
(c)	28	11,500	69,000	17.0	28	24,300	96,000	19.0
(d) C=15(lb/in^3)	28	13,200	97,000	6.0	28	26,200	119,000	8.0
(e) C=72(lb/in^3)	28	11,800	74,000	10.0	28	24,700	100,000	11.0
(f) C=240 (lb/in^3)	28	11,500	69,000	17.0	28	24,300	96,000	19.0
2. Free Earth Support	17.3	15,450	140,000	3.0	18.1	29,600	182,000	3.0
	10.5	13,600	108,000	6.0	11.2	27,700	142,000	6.0
3. "Equivalent Beam"	31.4	12,800	92,000	3.0	34.1	26,000	118,000	3.0
	17.8	12,700	90,000	6.0	19.4	25,900	117,000	6.0
4. Elastic Line Method	29.4	14,200	116,000	3.0	34.0	28,500	165,000	3.0
	19.4	12,500	86,000	6.0	19.7	26,500	125,000	6.0



K_o = Coefficient of Earth Pressure at Rest
K_p = Coefficient of Active Earth Pressure
P_h = Active Pressure due to the Backfill

FIG I PRESSURES ACTING ON THE EMBEDDED LENGTH OF WALL

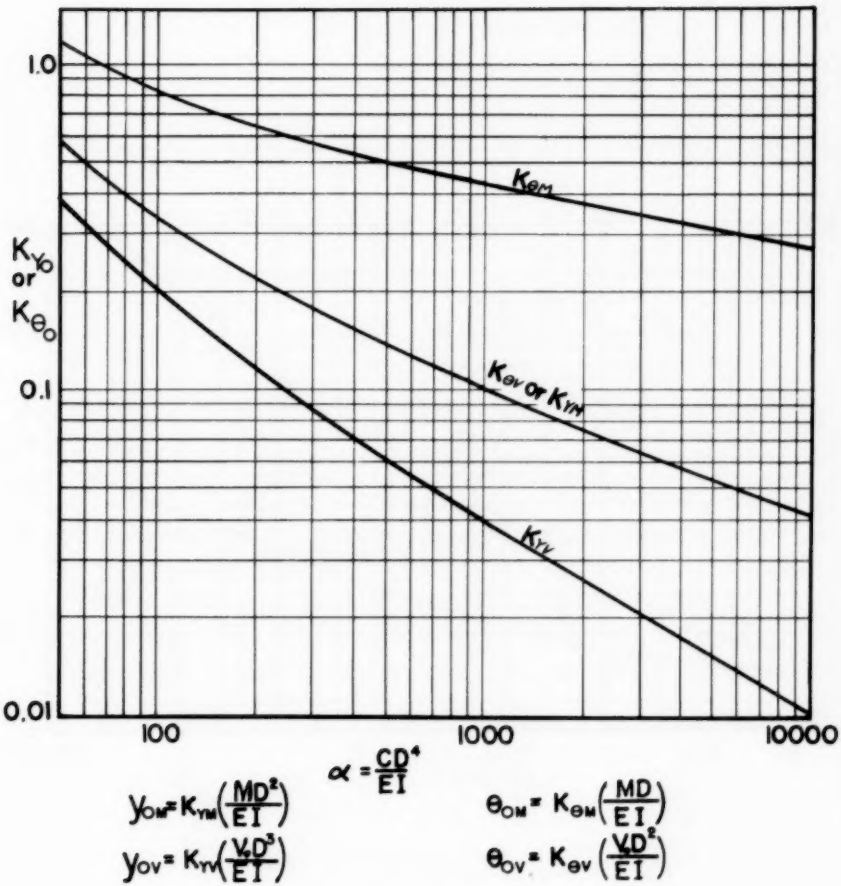


Fig. 2. Deflection and Slope Coefficients at the Dredge Line
(a) Due to End Moment and End Shear

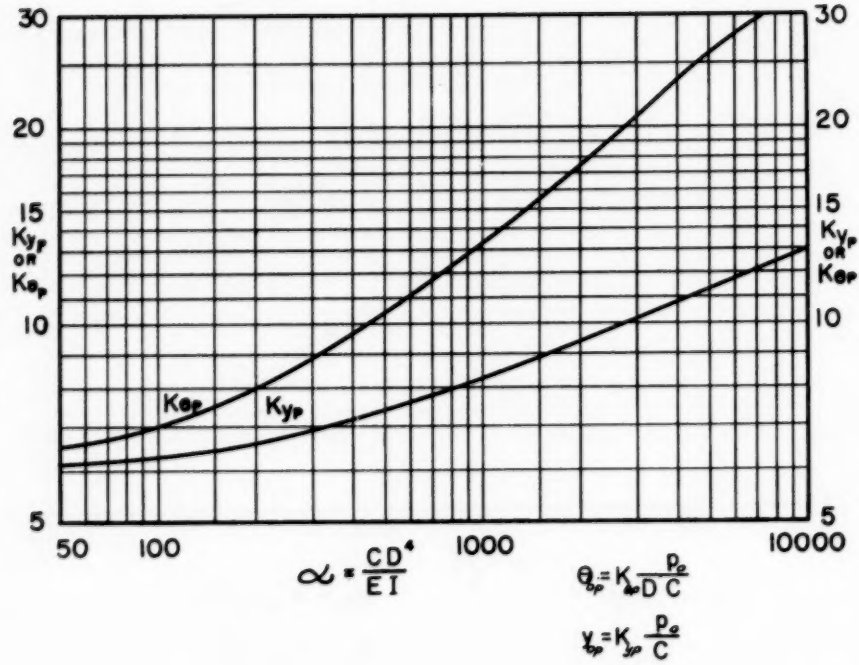


Fig. 2. Deflection and Slope Coefficients at the Dredge Line
(b) Due to Uniform Pressure

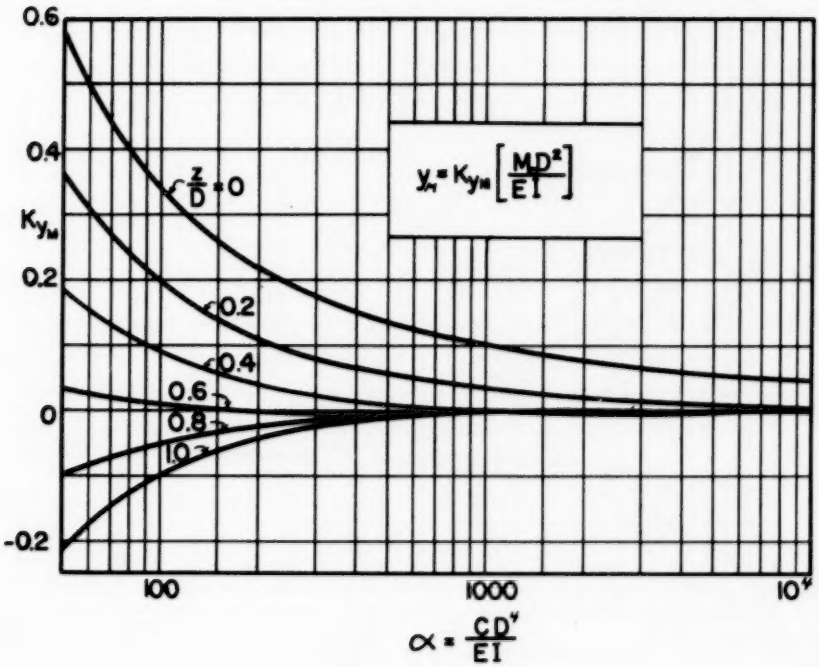


Fig. 3. Deflection Coefficients Due to End Moment Only

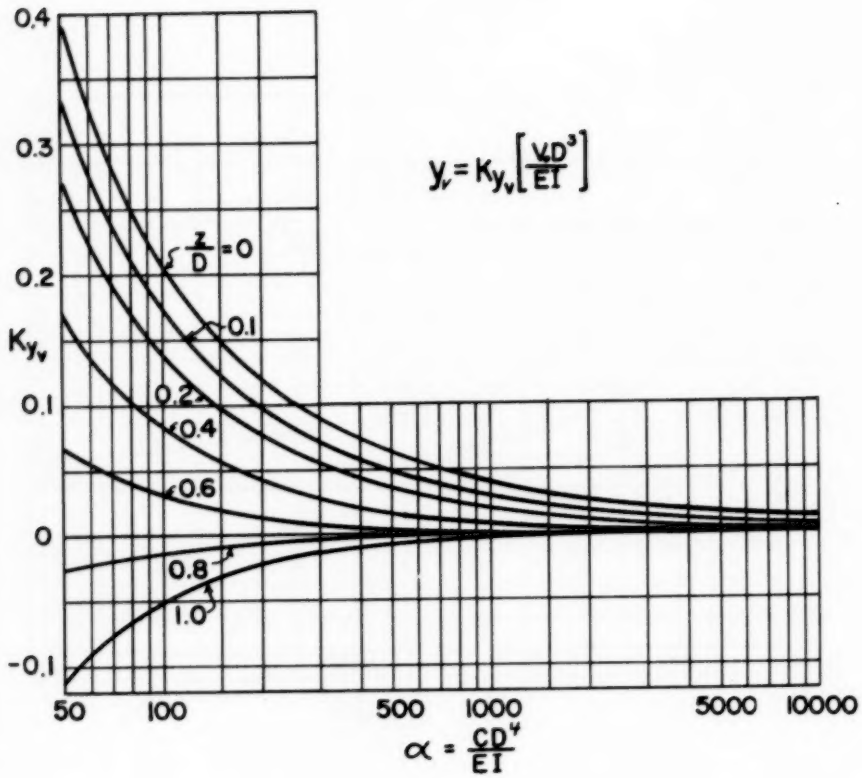


Fig. 4. Deflection Coefficients Due to End Shear Only

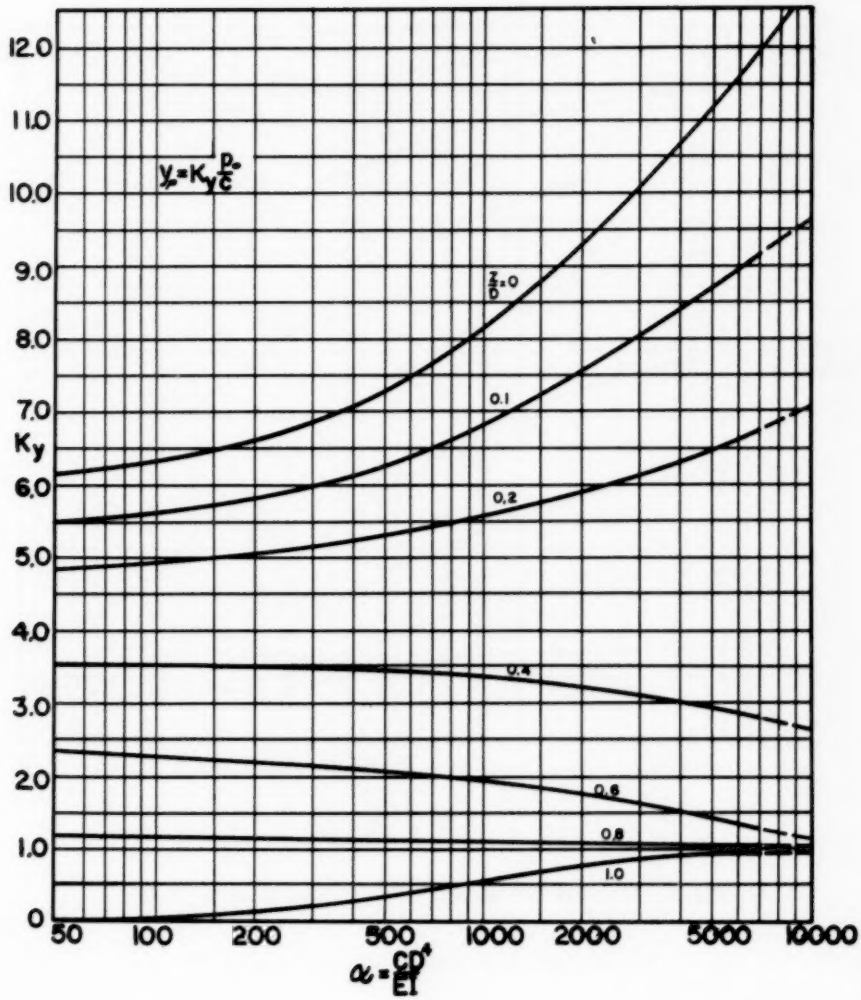


Fig. 5. Deflection Coefficients Due to Uniform Pressure Only

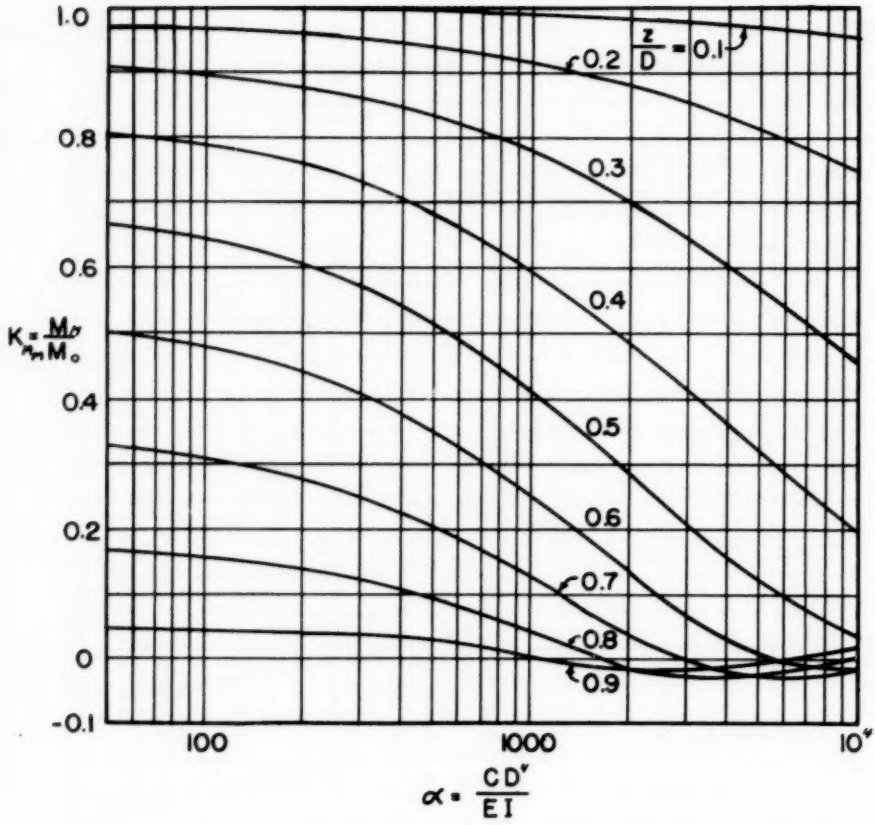


Fig. 6. Moment Coefficients Due to End Moment Only

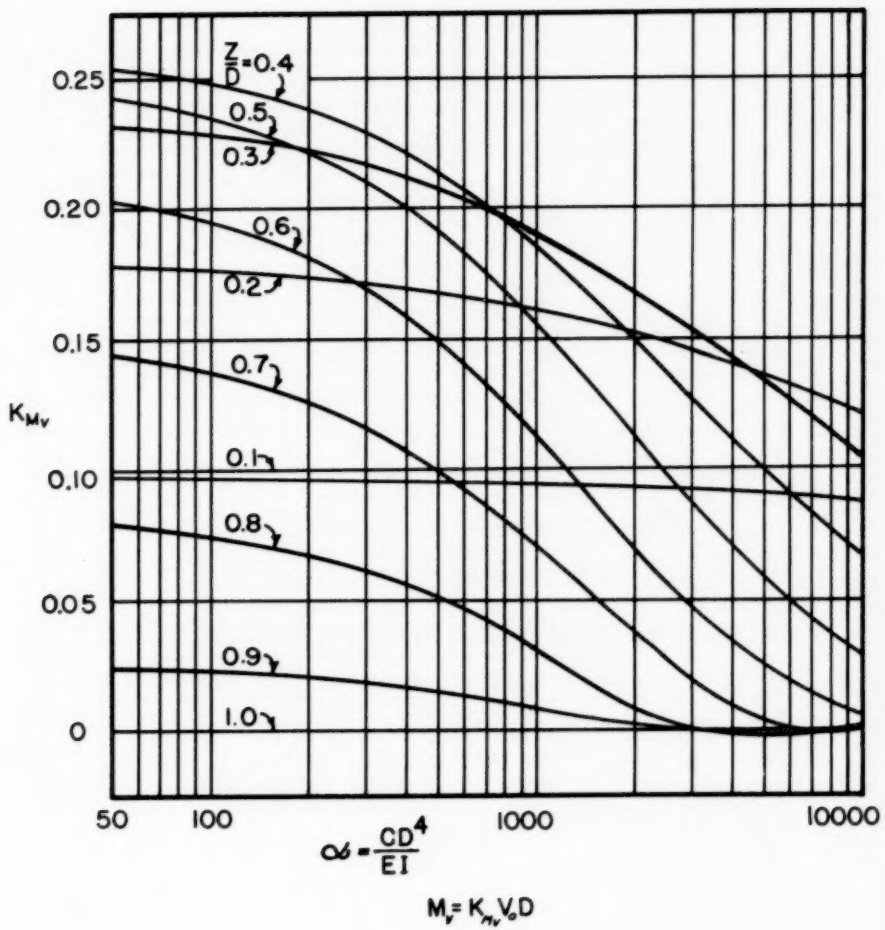


Fig. 7. Moment Coefficients Due to End Shear Only

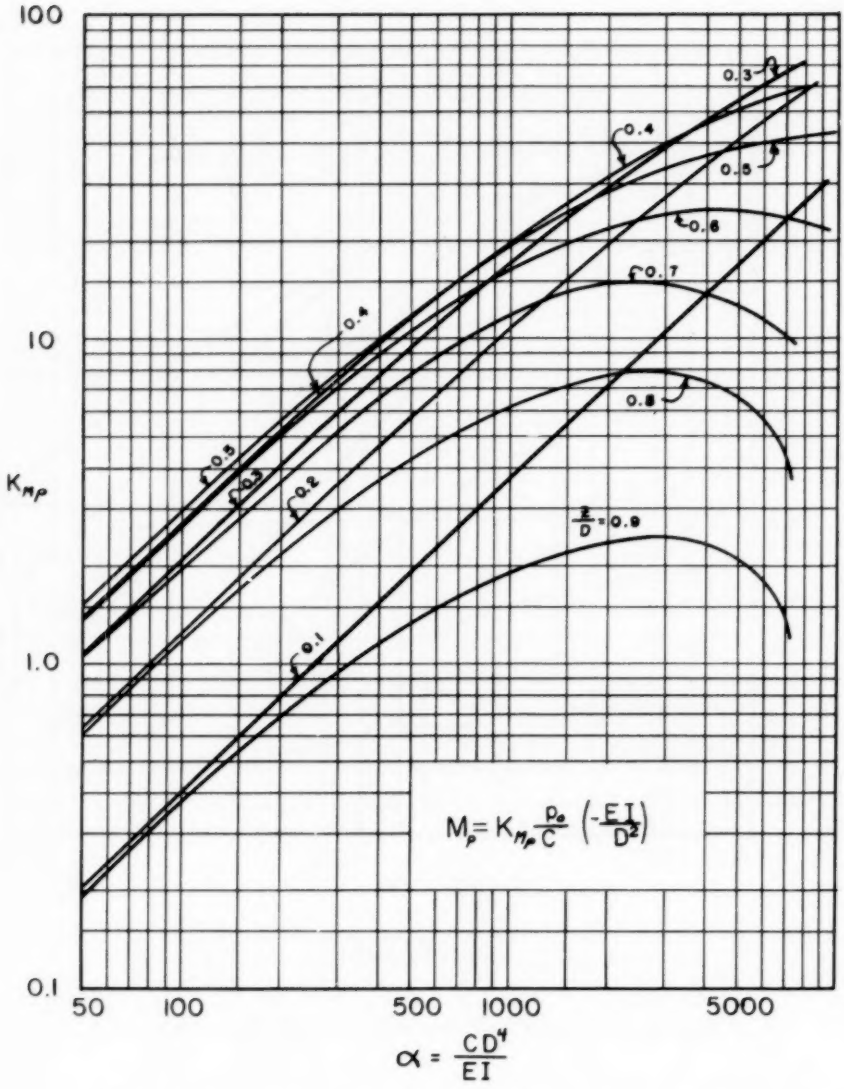


Fig. 8. Moment Coefficients Due to Uniform Pressure Only

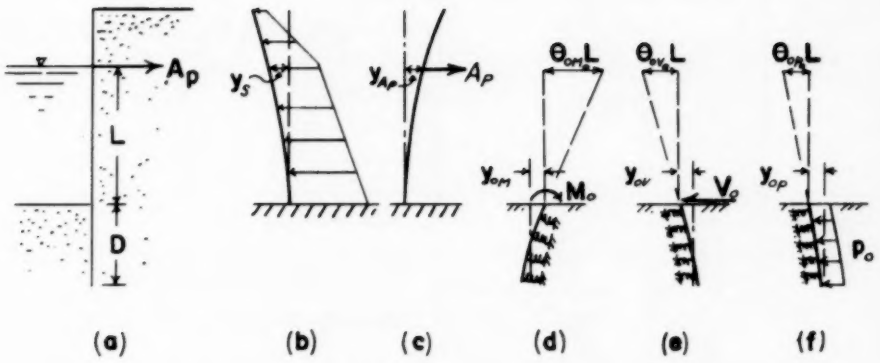
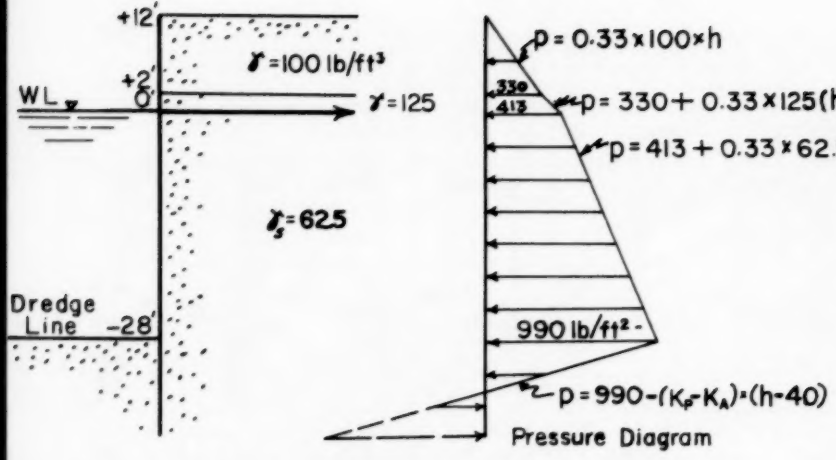
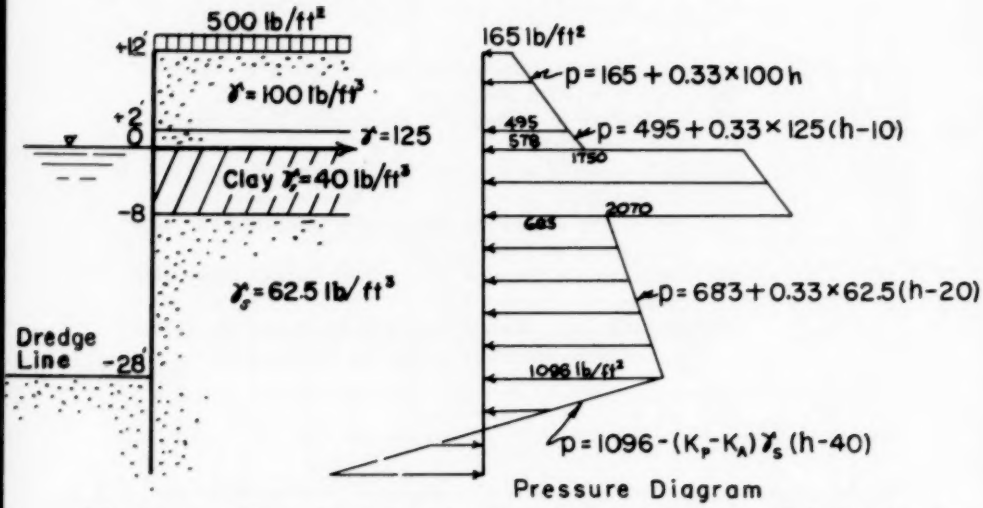


Fig. 9. Component Deflections of the Anchor Point
(a) Anchored Sheet Pile Wall
(b) Deflection of Pile Above the Dredge Line Due to Soil Load Only
(c) Deflection of Pile Above the Dredge Line Due to Anchor Pull
(d) Deflection at Dredge Line and at Anchor Point Due to Moment at Dredge Line
(e) Deflection at Dredge Line and at Anchor Point Due to Shear at Dredge Line
(f) Deflection at Dredge Line and at Anchor Point Due to Uniform Pressure below the Dredge Line

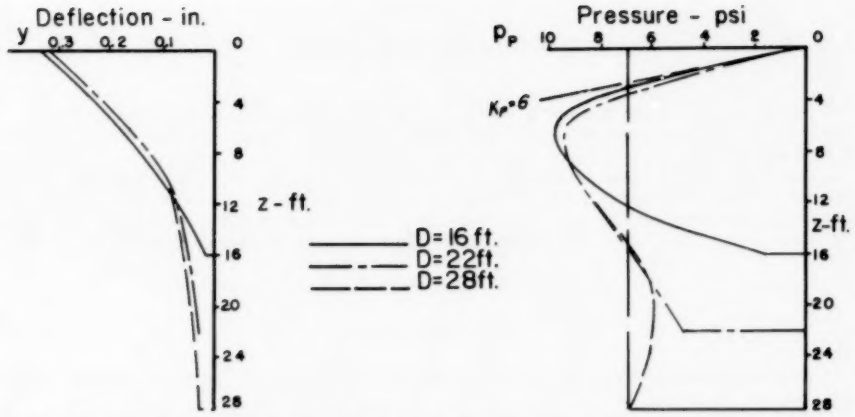


(a) Example A — Sand Backfill, $\phi = 30°$ No Surcharge

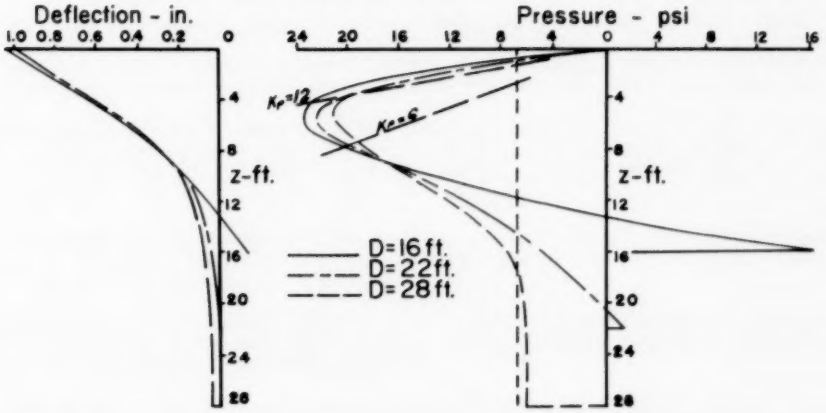


(b) Example B — Sand Backfill over Clay Layer, 500 lb/ft² Surcharge

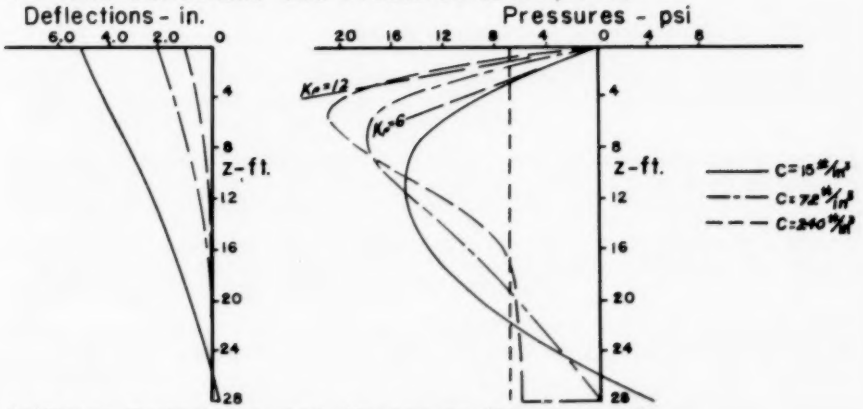
Fig. 10. Anchored Sheet Pile Wall Examples



(a) Deflections and Pressures Due to Uniform Load(Example A)



(b) Total Deflections and Pressures (Example A)



(c)Total Deflections and Pressures (Example A)

Fig. 11. Deflections and Pressures on Embedded Portion of Pile for Example A

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The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

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DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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